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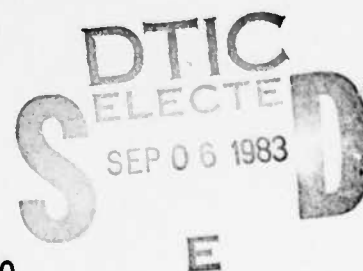


## On the Determination of the Index of Refraction of the Atmosphere

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## On the Determination of the Index of Refraction of the Atmosphere

### I. INTRODUCTION

It is the intent of this article to relate the index of refraction of optical and radio waves to the distribution of atmospheric species and to demonstrate that with today's technology (using rocket measurements of density, temperature, and winds) it is conceivable that the index of refraction in the mesosphere and stratosphere may be directly measured under turbulent conditions and compared to those values determined using incoherent radars and optical devices.

As in the Bean and Dutton<sup>1</sup> discussion, Debye<sup>2</sup> showed that molecular polarizability is composed of two effects: one due to the distortion of all the molecules by the impressed field and the other arising from an orientation effect exerted upon polar molecules. The polarization  $P$  of a polar liquid under the influence of a high-frequency radio field is given by Eq. (1).

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(Received for publication 23 February 1983)

1. Bean, B.R., and Dutton, E.J. (1966) Radio meteorology, Nat. Bur. Stand. Monograph 92.
2. Debye, P. (1957) Polar Molecules, Dover Publishing Co., New York.



$$P(\omega) = \frac{\epsilon - 1}{\epsilon + 2} \frac{M}{\rho} = \frac{4\pi N}{3} \left[ \alpha_0 + \frac{\mu^2}{3kT} \frac{1}{(1 + i\omega\tau)} \right] , \quad (1)$$

where

$\epsilon$  is the dielectric constant,

$M$  is the molecular weight,

$\rho$  is the density of the liquid,

$N$  is Avogadro's number,

$\alpha_0$  is the average polarizability of the molecules in the liquid assuming no interaction between molecules,

$\mu$  is the permanent dipole moment,

$k$  is Boltzmann's constant,

$T$  is the absolute temperature,

$\tau$  is the relaxation time required for external field-induced orientations of the molecules to return to random distribution after the field is removed, and

$\omega = 2\pi f$  where  $f$  is the frequency of the external field.

For frequencies less than 100 GHz,  $\omega\tau \ll 1$  so that Eq. (1) may be written

$$\frac{\epsilon - 1}{\epsilon + 2} \frac{M}{\rho} = \frac{4\pi N}{3} \left[ \alpha_0 + \frac{\mu^2}{3kT} \right] . \quad (2)$$

For non-polar gases ( $\mu = 0$ ), this equation reduces approximately to

$$\epsilon - 1 \cong 4\pi\alpha_0 \frac{\rho N}{M} . \quad (3)$$

Now from the definition of  $M$  and  $N$ , Eq. (3) becomes

$$\epsilon - 1 \cong 4\pi\alpha_0 n_a , \quad (4)$$

where  $n_a$  is the atmospheric number density.

With the inclusion of polar gases Eq. (2) becomes

$$(\epsilon - 1)_i \sim 4\pi n_{a_i} \left[ \alpha_{o_{a_i}} + \frac{\mu_{a_i}^2}{3kT} \right], \quad (5)$$

where  $n_{a_i}$  is the number density of species  $i$ .

For atmospheric gases, the value of the permeability ( $\mu$ ) may be assumed to be unity, with the exception of water vapor. Therefore,

$$\epsilon - 1 \approx 4\pi \left[ \sum_{i \neq H_2O} (n_{a_i} \alpha_i) + n_{H_2O} \left( \alpha_{H_2O} + \frac{\mu_{aH_2O}^2}{3kT} \right) \right]. \quad (6)$$

The equation for the index or refraction  $n_i \equiv \sqrt{\mu\epsilon}$  then equals

$$2(n_i - 1) \approx \mu\epsilon - 1 \approx 4\pi \left[ \sum_{i \neq H_2O} (n_{a_i} \alpha_i) + n_{H_2O} \left( \alpha_{H_2O} + \frac{\mu_{aH_2O}^2}{3kT} \right) \right] \quad (7)$$

or

$$N = (n_i - 1)10^6 = \left( \frac{\mu\epsilon - 1}{2} \right) \times 10^6$$

$$\cong 2\pi \times 10^6 \left[ \sum_i n_{a_i} \alpha_i + n_{H_2O} \frac{\mu_{aH_2O}^2}{3kT} \right], \quad (8)$$

where the summation now includes water vapor, and  $N$  is now the Dielectric constant.

To compare this relation to measurement, we examine the dielectric constant at one atmosphere using the density at ground level and the dielectric constant of  $O_2$  and  $N_2$ .<sup>3</sup> Thus,

$$(\epsilon - 1) \cdot 10^6 = 4\pi \times 10^6 (2.55 \times 10^{19})(\sim 1.7 \times 10^{-24}) = 545.$$

3. Banks, P.M., and Kockarts, G. (1973) Aeronomy, Academic Press.



This compares well to the value of 575 quoted in Table 1 of Bean and Dutton.<sup>1</sup> Thus, the dielectric constant  $N$ , as determined from Eq. (8), would be  $\sim 273$ , without the contribution of water vapor. Where compared to the measured value of 288, this result demonstrates that the index of refraction  $n_i$  or the dielectric constant  $N$ , as derived from basic considerations, are for the most part independent of temperature and a function of atmospheric species, with the exception of water vapor.

In the lower troposphere, water vapor is important in the determination of the dielectric constant through the second term in Eq. (8). However, in the upper troposphere, stratosphere, and mesosphere water vapor is not important to radio wave or optical scattering by neutrals and Eq. (8) becomes

$$N = (n_i - 1) \cdot 10^6 = (2\pi \times 10^6) \sum_i (n_{a_i} \alpha_i) \quad . \quad (9)$$

## 2. THE EFFECT OF TURBULENCE UPON OPTICAL AND RADIO WAVE SCATTERING

Given  $(n - 1) = C_1 \alpha n_a$  the fluctuation component can be defined as

$$n'_a = \bar{n}_a - n_a \quad ,$$

where

$\bar{n}_a$  is the ensemble average.

The turbulent adiabatic density fluctuation is thus defined as

$$\tilde{n}'_a = \tilde{\bar{n}}_a - \tilde{n}_a \quad ,$$

where

$$\tilde{n}_a \equiv \left[ n_a \left( \frac{P_0}{P} \right)^{1/\gamma} \right] \text{ is the potential density and}$$

$\gamma$  is the ratio of the specific heats.

Thus, it can be shown that

$$\begin{aligned} \frac{1}{\tilde{n}_a} \frac{\partial \tilde{n}_a}{\partial z} &= \frac{1}{n_a} \frac{\partial n_a}{\partial z} + \frac{mg}{\gamma kT} \\ &= \frac{1}{n_a} \left( \frac{\partial n_a}{\partial z} + \frac{n_a}{\gamma H_P} \right) \end{aligned} \quad (10)$$

where  $H_P$  is the atmospheric pressure scale height ( $\equiv kT/mg$ ). Thus, it is easy to show that the Brunt-Vaissalla frequency  $\omega_B$  can be expressed as:

$$\omega_B^2 = \frac{g}{\tilde{n}_a} \frac{\partial \tilde{n}_a}{\partial z} \equiv \frac{g}{\Theta} \frac{\partial \Theta}{\partial z} \quad (11)$$

where  $\Theta$  is potential temperature.

Given a turbulent region, the turbulent potential density "inertial" spectrum may be defined as

$$\langle (\tilde{n}'_a)^2 \rangle = \int_0^\infty \mathcal{E}_{n_a}(k) dk = \beta_1 \epsilon^{-1/3} \epsilon_{n_a} \int_0^\infty k^{-5/3} dk \quad (12)$$

where

- $\langle \rangle$  describe the ensemble average,
- $\mathcal{E}_{n_a}(k)$  is the scalar spectral energy density,
- $\epsilon$  is the rate of viscous dissipation,
- $\epsilon_n$  is the rate of decay of the density fluctuations,
- $k$  is the wavenumber, and
- $\beta_1$  is a constant.

Similarly, we can define the index of refraction fluctuation spectrum

$$\langle (n'_i)^2 \rangle = \int_0^\infty \mathcal{E}_{n_i}(k) dk = \beta_2 \epsilon^{-1/3} \epsilon_{n_i} \int_0^\infty k^{-5/3} dk \quad (13)$$

Making use of Eq. (8) we have (expressing all densities as potential densities without notation)

$$\langle n_i'^2 \rangle = (C_1 \alpha)^2 \langle n_a'^2 \rangle, \quad (14)$$

where

$C_1$  contains all the constants.

We may thus measure the amplitude of the turbulent index of refraction by the direct measurement of atmospheric density fluctuations. This is true for the optical and radio index of refraction in the lower atmosphere above the water vapor region and below  $\sim 55$  km for the radio index of refraction. The desired parameter to be measured is the quantity  $C_{n_i}^2$ , the coefficient of the spectrum, ( $\equiv \beta_2 \epsilon_{n_i} \epsilon^{-1/3}$ ), and from Eq. (14) we have  $C_{n_i}^2 = (C_1 \alpha)^2 C_{n_a}^2$ .

Consider now the equation for the production of turbulent heat flux  $\overline{w'\theta'}$  where  $w'$  is vertical turbulent velocity fluctuation, and  $\theta'$  is turbulent potential temperature fluctuation.

Ignoring dissipation, we have

$$\frac{\partial}{\partial t} \langle w'\theta' \rangle \approx 0 = \langle w'^2 \rangle \frac{\partial \theta}{\partial z} - \frac{\partial}{\partial z} \langle w'^2 \theta' \rangle - \left\langle \theta' \frac{\partial P'}{\partial z} \right\rangle + \frac{g}{\Theta} \langle \theta'^2 \rangle, \quad (16)$$

where  $\Theta$  is the mean atmospheric potential temperature. If the turbulent transfer and pressure terms are small and in opposition, we have

$$\langle \theta'^2 \rangle \cong \frac{\Theta}{g} \frac{\partial \Theta}{\partial z} \langle w'^2 \rangle. \quad (17)$$

In spectral form, we then have

$$\langle \theta'^2 \rangle = \beta_3 \epsilon_\theta \epsilon^{-1/3} \int_0^\infty k^{-5/3} dk \cong \frac{\Theta}{g} \frac{\partial \Theta}{\partial z} \alpha_1 \epsilon^{2/3} \int_0^\infty k^{-5/3} dk \quad (18)$$

or equating coefficients,

$$\beta_3 \epsilon_\theta \epsilon^{-1/3} \cong \frac{\Theta}{g} \frac{\partial \Theta}{\partial z} \epsilon^{2/3}$$

and

$$C_{\theta}^2 \cong \alpha_1 \frac{\Theta}{g} \frac{\partial \Theta}{\partial z} \epsilon^{2/3} \quad , \quad (19)$$

where

$\beta_3$  is a constant of proportionality,

$\epsilon_{\theta}$  is the dissipation rate of temperature fluctuations by molecular diffusion,

$\langle (\theta')^2 \rangle$  is the mean square temperature fluctuation, and

$C_{\theta}^2$  is the representation of the coefficients, similar to  $C_n^2$ , and

$\alpha_1$  is the constant of proportionality for the vertical component of the turbulent velocity.

Similar logic can be applied to  $\langle (n'_a)^2 \rangle$ , the neutral density fluctuation giving

$$\langle (n'_a)^2 \rangle \cong \frac{\tilde{n}_a}{g} \frac{\partial \tilde{n}_a}{\partial z} \langle (w')^2 \rangle \quad (19)$$

and, for an inertial velocity spectrum,

$$\langle (n'_a)^2 \rangle \cong \frac{\tilde{n}_a}{g} \frac{\partial \tilde{n}_a}{\partial z} \alpha_1 \epsilon^{2/3} \int_0^{\infty} k^{-5/3} dk \quad . \quad (20)$$

Let us now consider the component  $C_{n_i}^2$ . This quantity, by definition, the refractivity structure constant, is given by Tatarskii<sup>4</sup> as (Balsley and Gage<sup>5</sup>)

$$C_{n_i}^2 = 5.45 \langle (n'_i)^2 \rangle L_o^{-2/3} \quad , \quad (21)$$

where  $L_o$  is the outer length scale of the turbulent spectra. Then the reflectivity ( $\eta$ ) for backscatter from turbulent fluctuations by the neutral density is (Balsley and Gage<sup>5</sup>)

$$\eta = 0.38 C_{n_i}^2 \lambda^{-1/3} \quad , \quad (22)$$

4. Tatarskii, V.I. (1971) The effects of the turbulent atmosphere on wave propagation, U.S. Dept. Commerce, pp. 74-76.

5. Balsley, B.B., and Gage, K.S. (1980) The MST radar technique, Rev. Pure Appl. Geophys. 118:452.

where  $\lambda$  is the radar wavelength. This  $\langle C_{n_i}^2 \rangle$  however is nothing more than the coefficient of the spectrum of the index of refraction fluctuations, and for a passive neutral atmosphere, it can be expressed as

$$C_{n_i}^2 = \beta_2 \epsilon_{n_i} \epsilon^{-1/3}, \text{ and using (14) and (20)}$$

$$C_{n_i}^2 = (C_1 \alpha)^2 C_{n_a}^2 = 4\pi \alpha_i^2 \alpha_l \frac{\tilde{n}_a}{g} \frac{\partial \tilde{n}_a}{\partial z} \epsilon^{2/3}. \quad (23)$$

Substituting into Eq. (22) we then have

$$\eta \cong (0.38) (4\pi^2) \alpha^2 \alpha_l \frac{\tilde{n}_a}{g} \frac{\partial \tilde{n}_a}{\partial z} \epsilon^{2/3}, \quad (24)$$

which directly relates the reflectivity to the dissipation. Simplifying the result by substituting the value of the molecular polarizability and constants (in the Gaussian mks system), we have

$$\tilde{n}_i = 1.068 \times 10^{-29} \tilde{n}_a.$$

Thus,

$$C_{n_i}^2 = (1.068 \times 10^{-29})^2 C_{n_a}^2,$$

$$\therefore C_{n_i}^2 = (1.068 \times 10^{-29})^2 \alpha_l \frac{\tilde{n}_a}{g} \frac{\partial \tilde{n}_a}{\partial z} \epsilon^{2/3}, \quad (25)$$

where we have used  $\bar{\alpha} = 1.7 \times 10^{-30} \text{ m}^3$ .

The question now is how to determine the neutral index of refraction. Obviously one method is by direct measurement of atmospheric density to fine enough resolution so that we can determine the local regions where turbulence is creating these fine scale fluctuations of temperature, velocity, and density. Also obviously, it would be a prohibitive venture in terms of cost and time to acquire these data to the necessary accuracy for this project. Therefore, to determine probable estimates of the turbulent contribution to the local index of refraction, we must utilize existing bodies of data that contain winds, temperature, and density measured to a fairly fine altitude resolution. The only existing sequential data over a large

enough area and altitude span are the "Meteorological Rocket Network" and the "Rawinsonde" data bases. However, given these data, the next question is how to determine the probable turbulent contribution to the index of refraction, since these data in the troposphere, stratosphere and mesosphere have a coarse altitude resolution compared to the turbulent fluctuation scale. There have been two approaches to this problem.\* These have been formulated (1) to determine the turbulence contribution to the index of refraction observed by radars (see the reference list in Balsley and Gage<sup>5</sup>) in the troposphere and stratosphere, and (2) to determine directly the turbulent parameters at local atmospheric sites, given the condition that the local atmosphere is in a state of destabilization as determined by the calculation of the local Richardson number. It is to this second aspect that we will now address our efforts.

Zimmerman and Murphy,<sup>6</sup> assuming that Deacon's<sup>7</sup> analysis of the boundary layer measurements of the ratio ( $w'/U$ ) (where  $w'$  is the vertical turbulent velocity and  $U$  is the mean wind velocity) as a function only of Richardson number could be extrapolated from the boundary layer to the upper atmosphere, determined the in-situ turbulent intensities, dissipation rates, and diffusivities that seemed quite reasonable when compared to other measurements of these parameters. Subsequently, Zimmerman and Keneshea<sup>8</sup> reasoned that the energy balance equation would be a more accurate representation of turbulence and probably would be a more altitude invariant formulation than the vertical turbulent to horizontal mean velocity ratio of Deacon.<sup>7</sup> Using this reasoning they determined not only the diffusivities and dissipation rates, but also the local heat and momentum transfer rates. Comparing these analyses (Figure 1) to that of Ebel,<sup>9</sup> who had deduced the required momentum and heat sources and sinks from the Groves<sup>10</sup> model atmosphere, they found quite good agreement of both the phase and amplitude required by Ebel. In a more recent work Zimmerman and Keneshea,<sup>11</sup> demonstrate

\* This does not imply that these are the only approaches, since there is a significant body of reports on the diffraction of optical wavelength by the atmosphere.

6. Zimmerman, S. P., and Murphy, E. A. (1971) Stratospheric and Mesospheric Turbulence, Dynamic and Chemical Coupling, Reidel Publishing, Dordrecht, Holland.
7. Deacon, E. L. (1959) Inter. J. An Pollution 2:92.
8. Zimmerman, S. P., and Keneshea, T. J. (1981) Turbulent heating and transfer in the stratosphere and upper mesosphere, Handbook for MAP, vol 2, University of Illinois Press.
9. Ebel, A. (1974) Tellus XXVI:325.
10. Groves, G. V. (1969) Wind models from 60 to 130 km altitude for different months and latitudes, J. Brit. Interplanet. Soc. 22:285-307.
11. Zimmerman, S. P., and Keneshea, T. J. (1983) Turbulent heating and transfer in the stratosphere and mesosphere (submitted to J. Atmos. Terr. Phys.)

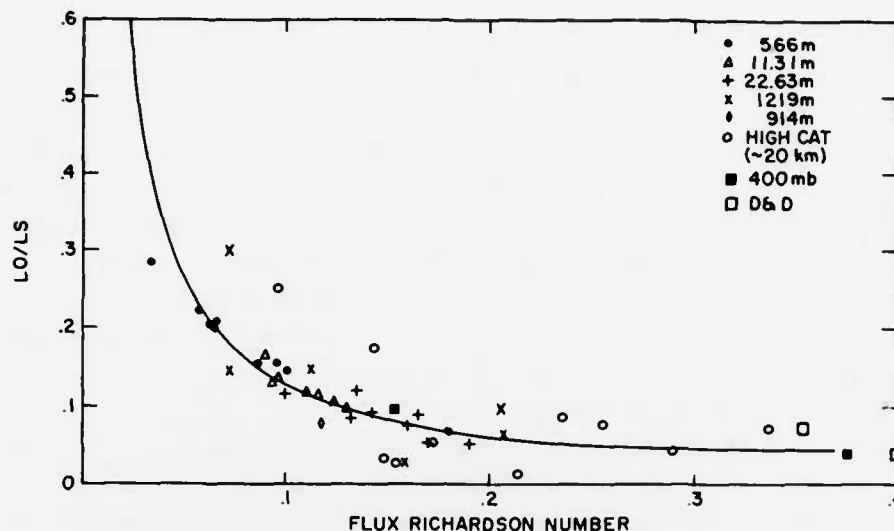


Figure 1. The Normalized, Non-dimensional Heat Flux  $\overline{(w'\theta')}$  Comparing the Data From 5.66 m to 20 km

that indeed a non-dimensional heat flux and Reynolds Stress may be determined from the boundary layer data. This non-dimensional heat flux is shown to be fairly invariant with altitude (Figure 2), at least to 20 km. Thus, following that work we can determine the fluctuation components necessary to explain the local index of refraction effects due to turbulence in the stratosphere and lower mesosphere, if we accept the logic that we can extrapolate this non-dimensional heat flux and Reynolds Stress to the higher altitudes. The determinations of  $C_{n_i}^2$  using Eqs. (21) and (23) are that from Tatarskii's analysis and that suggested here, respectively.

From Tatarskii

$$C_{n_i}^2 = 5.45 \langle (n_i')^2 \rangle L_0^{-2/3} ,$$

where  $L_0$  is determined from the spectral equations, that is,

$$\langle q'^2 \rangle = \alpha_T \epsilon^{2/3} \int_{k_0}^{\infty} k^{-5/3} dk \cong \frac{3}{2} \alpha \epsilon^{2/3} k_0^{-2/3} , \quad (26)$$

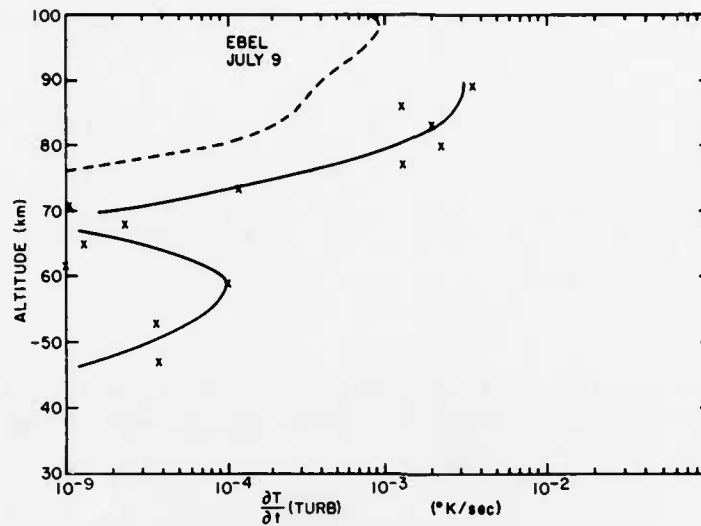


Figure 2. The Annual Average of the Divergence of the Turbulent Heat Flux Added to the Rate of Dissipation

where

$$k_o^{-2/3} = L_o^{2/3} = \frac{2}{3} \frac{\langle q'^2 \rangle}{\alpha_T \epsilon^{2/3}}$$

where  $q'$  is turbulent velocity and  $w'$ ,  $v'$  and  $u'$  are the components. Thus,

$$L_o = \left( \frac{2}{3} \right)^{3/2} \frac{\langle q'^2 \rangle^{3/2}}{\epsilon}$$

and  $\langle (n'_i)^2 \rangle$  is determined from

$$\langle (n'_i)^2 \rangle = (C\alpha)^2 \langle (n'_{ai})^2 \rangle = (C\alpha)^2 \frac{\langle (\theta')^2 \rangle}{\Theta^2} n_a^2 \quad (27)$$

12. Groves, G.V. (1971) Atmospheric Structure and Its Variations in the Region From 25 to 120 km, AFCRL-64-0410, AD 737794.



Therefore,

$$C_{n_i}^2 = (1.068 \times 10^{-29})^2 \cdot 5.45 \frac{\langle (\theta')^2 \rangle}{\Theta^2} \tilde{n}_a^2 .$$

Relation (23) simply states

$$C_{n_i}^2 = (1.068 \times 10^{-29})^2 \alpha_{\perp} \frac{\tilde{n}}{g} \frac{\partial \tilde{n}}{\partial z} \epsilon^{2/3} ,$$

where  $\epsilon$  is determined from the energy balance. Table 1 displays the input conditions and results of using the above two relations on data from the Meteorologic Rocket Network.

Table 1. The Parameters for This Work as Compared to Tatarskii's<sup>4</sup>

	This Work				Tatarskii		
	$\tilde{n} \text{ (m}^{-3}\text{)}$	$\partial \tilde{n} / \partial z \text{ (m}^{-4}\text{)}$	$\epsilon \left( \frac{\text{m}^2}{\text{sec}^3} \right)$	$C_{n_i}^2 \text{ (m}^{-2}\text{)}$	$\tilde{n} \text{ (m}^{-3}\text{)}$	$L_o \text{ (m)}$	$C_{n_i}^2 \text{ (m}^{-2}\text{)}$
1.	$2.23^{24}$	$6.19^{18}$	1.94	$1.78^{-16}$	$7^{21}$	$1.92^2$	$1.56^{-16}$
2.	$1.8^{24}$	$1.7^{19}$	$2.6^{-2}$	$2.56^{-17}$	$1.96^{21}$	$1.35^2$	$1.7^{-17}$

Note: The power relation is  $10^P$ .

As observed, both methods compare quite well, but Eq. (23) is the simpler relation to work with and easier to determine.

Future work will be to analyze the existing data base to determine probability factors of  $C_{n_i}^2$  as functions of altitude and time by turbulence per se. Later work would be to supplement these data with results from analyses using the Van Zandt et al formulation<sup>13</sup> for the probable distribution of turbulence given measurements of Richardson number that indicate a non-turbulent volume.

13. Van Zandt, T.E., Gage, K.S., and Warnock, J.M. (1981) An improved model for the calculations of profiles of  $C_{n_i}^2$  and  $\epsilon$  in the field atmosphere from background profiles of wind, temperature, and humidity, 20th Conference on Radar Meteorology, Boston, Mass.

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